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## Physics Letters B

[www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)Extraction of the CP-violating phase  $\gamma$  using  $B \rightarrow K\pi\pi$  and  $B \rightarrow KK\bar{K}$  decaysBhubanjyoti Bhattacharya<sup>a</sup>, Maxime Imbeault<sup>b</sup>, David London<sup>a,\*</sup><sup>a</sup> Physique des Particules, Université de Montréal, C.P. 6128, succ. centre-ville, Montréal, QC, H3C 3J7, Canada<sup>b</sup> Département de physique, Cégep de Saint-Laurent, 625, avenue Sainte-Croix, Montréal, QC, H4L 3X7, Canada

## ARTICLE INFO

## Article history:

Received 10 October 2013

Received in revised form 12 November 2013

Accepted 15 November 2013

Available online 22 November 2013

Editor: M. Cvetič

## ABSTRACT

Using the BABAR measurements of the Dalitz plots for  $B^0 \rightarrow K^+\pi^0\pi^-$ ,  $B^0 \rightarrow K^0\pi^+\pi^-$ ,  $B^+ \rightarrow K^+\pi^+\pi^-$ ,  $B^0 \rightarrow K^+K^0K^-$ , and  $B^0 \rightarrow K^0K^0\bar{K}^0$  decays, we demonstrate that it is possible to cleanly extract the weak phase  $\gamma$ . We find four possible solutions. Three of these –  $32^\circ$ ,  $259^\circ$ , and  $315^\circ$  – are in disagreement with the SM, while one –  $77^\circ$  – is consistent with the SM. An advantage of this Dalitz-plot method is that one can obtain many independent measurements of  $\gamma$ , thereby reducing its statistical error. An accurate determination of the errors, however, requires detailed knowledge of the data.

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One of the main aims of  $B$  physics is to test the standard model (SM) explanation of CP-violation, which is that it is due to a complex phase in the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix. To this end, one measures the three angles of the unitarity triangle [1],  $\alpha$ ,  $\beta$  and  $\gamma$ , in many different ways, and looks for discrepancies.

The conventional wisdom has been that one can cleanly extract CKM phase information only from two-body  $B$  decays. However, it was recently shown that, contrary to this point of view, such information can also be obtained from charmless three-body  $B$  decays [2,3]. Based on this result, a method was proposed for extracting the weak phase  $\gamma$  from  $B \rightarrow K\pi\pi$  and  $B \rightarrow KK\bar{K}$  decays [4]. Specifically,  $\gamma$  is obtained by combining information from the Dalitz plots for  $B^0 \rightarrow K^+\pi^0\pi^-$ ,  $B^0 \rightarrow K^0\pi^+\pi^-$ ,  $B^+ \rightarrow K^+\pi^+\pi^-$ ,  $B^0 \rightarrow K^+K^0K^-$ , and  $B^0 \rightarrow K^0K^0\bar{K}^0$ .

In this Letter, we apply this method to experimental data. We use the measurements of the Dalitz plots of the five  $B \rightarrow K\pi\pi$  and  $B \rightarrow KK\bar{K}$  decays by the BABAR Collaboration [5]. One key point is that this method for extracting  $\gamma$  in fact applies to each point in the Dalitz plot. However, the value of  $\gamma$  is independent of momentum, so that the method really represents many independent measurements of  $\gamma$ . A preliminary analysis presented in Ref. [6] considered a naive average over all such measurements. In this

Letter we improve upon this and perform a combined likelihood fit to extract  $\gamma$  from multiple Dalitz-plot points.

We begin by briefly reviewing the principal results of Refs. [2,3]. There are three ingredients that permit the extraction of weak phases from three-body charmless  $B$  decays. First, the decay amplitudes can be expressed in terms of diagrams. These are similar to those of two-body  $B$  decays [7], except that here it is necessary to “pop” a quark pair from the vacuum. The three-body diagrams are described in detail in Ref. [2]. (As we consider  $\bar{b} \rightarrow \bar{s}$  transitions, the  $B^+$  decay amplitude can receive a contribution from the annihilation diagram. This is neglected.) Note that, unlike the two-body diagrams, the three-body diagrams are momentum dependent.

Second, it is possible to fix the symmetry of the final state. This is done using the Dalitz plot of  $B \rightarrow P_1P_2P_3$  (the  $P_i$  are pseudoscalar mesons) [2]. Denoting by  $p_i$  the momentum of each  $P_i$ , one defines the three Mandelstam variables  $s_{ij} \equiv (p_i + p_j)^2$ . These are not independent, but obey  $s_{12} + s_{13} + s_{23} = m_B^2 + m_1^2 + m_2^2 + m_3^2$ . Now, the Dalitz plot is given in terms of two Mandelstam variables, say  $s_{12}$  and  $s_{13}$ . The key point is that the experimental Dalitz-plot analysis allows one to reconstruct the decay amplitude  $\mathcal{M}(B \rightarrow P_1P_2P_3)(s_{12}, s_{13})$ . The amplitude for a state with a given symmetry is then found by applying this symmetry to  $\mathcal{M}(s_{12}, s_{13})$ . This amplitude is used to compute all (momentum-dependent) observables for the decay. For example, the final state  $K_S\pi^+\pi^-$  has CP + if the  $\pi^+\pi^-$  pair is symmetrized. The amplitude for this state is  $[\mathcal{M}(s_{12}, s_{13}) + \mathcal{M}(s_{13}, s_{12})]/\sqrt{2}$ .

Third, in Ref. [3] it was shown that, as is the case in two-body decays [8], under flavor SU(3) there are relations between the electroweak penguin (EWP) and tree diagrams for  $\bar{b} \rightarrow \bar{s}$  transitions. These take the simple form

$$P'_{EWi} = \kappa T'_i, \quad P'^C_{EWi} = \kappa C'_i \quad (i = 1, 2);$$

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$$\kappa \equiv -\frac{3}{2} \frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|} \frac{c_9 + c_{10}}{c_1 + c_2}, \quad (1)$$

where the  $c_i$  are Wilson coefficients and  $\lambda_p^{(s)} = V_{pb}^* V_{ps}$ . Note: the EWP-tree relations hold only for the state that is fully symmetric under exchanges of the final-state particles. However, the amplitude for this state can be found as described above using the Dalitz plot:

$$\mathcal{M}_{\text{fs}} = \frac{1}{\sqrt{6}} [\mathcal{M}(s_{12}, s_{13}) + \mathcal{M}(s_{13}, s_{12}) + \mathcal{M}(s_{12}, s_{23}) + \mathcal{M}(s_{23}, s_{12}) + \mathcal{M}(s_{23}, s_{13}) + \mathcal{M}(s_{13}, s_{23})], \quad (2)$$

where the subscript “fs” stands for “fully symmetric.”

We now describe the method proposed in Ref. [4] for extracting the weak phase  $\gamma$  from  $B \rightarrow K\pi\pi$  and  $B \rightarrow KK\bar{K}$  decays. The following five decays are considered:  $B^0 \rightarrow K^+\pi^0\pi^-$ ,  $B^0 \rightarrow K^0\pi^+\pi^-$ ,  $B^+ \rightarrow K^+\pi^+\pi^-$ ,  $B^0 \rightarrow K^+K^0K^-$ , and  $B^0 \rightarrow K^0K^0\bar{K}^0$ . In writing the amplitudes for these five processes in terms of diagrams, we note the following. For  $B \rightarrow K\pi\pi$  decays, the quark pair popped from the vacuum is  $u\bar{u}$  or  $d\bar{d}$  (under isospin, these diagrams are equal), while the  $B \rightarrow KK\bar{K}$  decays may have a popped  $s\bar{s}$  pair. Now, the imposition of the EWP-tree relations assumes flavor-SU(3) symmetry. But this also implies that diagrams with a popped  $s\bar{s}$  quark pair are equal to those with a popped  $u\bar{u}$  or  $d\bar{d}$ . In other words, under flavor-SU(3) symmetry the diagrams in  $B \rightarrow KK\bar{K}$  decays are the same as those in  $B \rightarrow K\pi\pi$  decays.

Note, however, that flavor-SU(3) symmetry is not exact. It is therefore important to keep track of a possible difference between  $B \rightarrow K\pi\pi$  and  $B \rightarrow KK\bar{K}$  decays.

The amplitudes for the five decays in terms of diagrams are given in Ref. [4]. We define the following four effective diagrams:

$$a \equiv -\tilde{P}'_{tc} + \kappa \left( \frac{2}{3} T'_1 + \frac{1}{3} C'_1 + \frac{1}{3} C'_2 \right), \\ b \equiv T'_1 + C'_2, \quad c \equiv T'_2 + C'_1, \quad d \equiv T'_1 + C'_1. \quad (3)$$

The decay amplitudes can now be written in terms of five diagrams,  $a$ – $d$  and  $\tilde{P}'_{uc}$ :

$$2A(B^0 \rightarrow K^+\pi^0\pi^-)_{\text{fs}} = be^{i\gamma} - \kappa c, \\ \sqrt{2}A(B^0 \rightarrow K^0\pi^+\pi^-)_{\text{fs}} = -de^{i\gamma} - \tilde{P}'_{uc}e^{i\gamma} - a + \kappa d, \\ \sqrt{2}A(B^+ \rightarrow K^+\pi^+\pi^-)_{\text{fs}} = -ce^{i\gamma} - \tilde{P}'_{uc}e^{i\gamma} - a + \kappa b, \\ \sqrt{2}A(B^0 \rightarrow K^+K^0K^-)_{\text{fs}} = \alpha_{\text{SU}(3)}(-ce^{i\gamma} - \tilde{P}'_{uc}e^{i\gamma} - a + \kappa b), \\ A(B^0 \rightarrow K^0K^0\bar{K}^0)_{\text{fs}} = \alpha_{\text{SU}(3)}(\tilde{P}'_{uc}e^{i\gamma} + a), \quad (4)$$

where  $\alpha_{\text{SU}(3)}$  measures the amount of flavor-SU(3) breaking.

In the flavor-SU(3) limit ( $|\alpha_{\text{SU}(3)}| = 1$ ),  $A(B^+ \rightarrow K^+\pi^+\pi^-)_{\text{fs}} = A(B^0 \rightarrow K^+K^0K^-)_{\text{fs}}$ , so that the  $B^+$  decay does not furnish any new information. The remaining four amplitudes depend on ten theoretical parameters: five magnitudes of diagrams, four relative strong phases, and  $\gamma$ . But in principle experiment can measure eleven observables: the decay rates and direct CP asymmetries for the four  $B^0$  decays, and the indirect CP asymmetries of  $B^0 \rightarrow K^0\pi^+\pi^-$ ,  $B^0 \rightarrow K^+K^0K^-$  and  $B^0 \rightarrow K^0K^0\bar{K}^0$ . With more observables than theoretical parameters,  $\gamma$  can be extracted from a fit. Furthermore, if one allows for SU(3) breaking ( $|\alpha_{\text{SU}(3)}| \neq 1$ ), we can add two more observables: the decay rate and direct CP asymmetry for the  $B^+$  decay. In this case it is possible to extract  $\gamma$  even with the inclusion of  $|\alpha_{\text{SU}(3)}|$  as a fit parameter.

As has been stressed above, the diagrams and observables are both momentum dependent. Thus, the extraction of  $\gamma$  can be performed at each point in the Dalitz plot. However, the value of  $\gamma$  is

independent of momentum, so that we really have *many* independent measurements of  $\gamma$  (up to experimental correlations between different parts of the Dalitz plot). When these are appropriately combined, the statistical error can be reduced.

We are now in a position to apply this method for extracting  $\gamma$  to real experimental data. BABAR has measured the Dalitz plots of the five  $B \rightarrow K\pi\pi$  and  $B \rightarrow KK\bar{K}$  decays [5]. The first step in performing a fit is to collect the observables. This is done as follows. An isobar model is used to analyze the three-body Dalitz plots. Here the decay amplitude is expressed as the sum of a non-resonant and several intermediate resonant contributions:

$$\mathcal{M}(s_{12}, s_{13}) = \mathcal{N}_{\text{DP}} \sum_j c_j e^{i\theta_j} F_j(s_{12}, s_{13}), \quad (5)$$

where the index  $j$  runs over all contributions. Each contribution is expressed in terms of isobar coefficients  $c_j$  (magnitude) and  $\theta_j$  (phase), and a dynamical wave function  $F_j$ .  $\mathcal{N}_{\text{DP}}$  is a normalization constant. The  $F_j$  take different forms depending on the contribution. The  $c_j$  and  $\theta_j$  are extracted from a fit to the Dalitz-plot event distribution. With the amplitude in hand, the observables can be constructed at each point in the Dalitz plot, and a fit can then be performed.

Such isobar analyses were performed by BABAR for each of the five three-body decays of interest [5]. The isobar coefficients found, together with their assumed wave functions ( $F_j$ ), allow us to reconstruct the amplitude for each three-body decay as a function of the relevant Mandelstam variables. We have chosen the normalization constant such that the integral of  $|\mathcal{M}|^2$  over the kinematically-allowed Dalitz-plot phase space gives the experimental branching fraction ( $\mathcal{B}_{\text{Exp}}$ ). We then construct  $\mathcal{M}_{\text{fs}}$  using Eq. (2). This process is repeated with the information available for the CP-conjugate process, where we construct  $\bar{\mathcal{M}}_{\text{fs}}$ . The experimental observables are then obtained as follows:

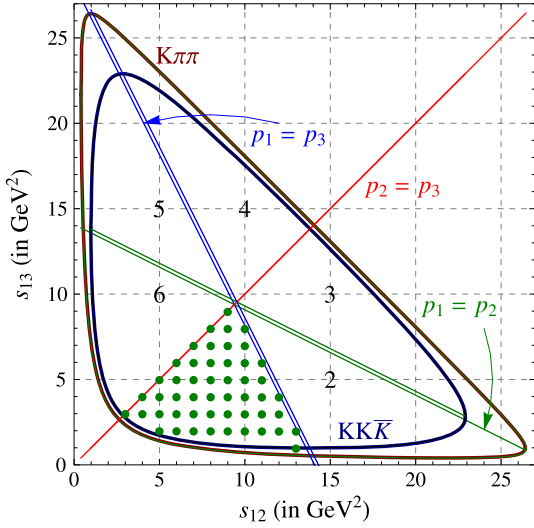
$$X(s_{12}, s_{13}) = |\mathcal{M}_{\text{fs}}(s_{12}, s_{13})|^2 + |\bar{\mathcal{M}}_{\text{fs}}(s_{12}, s_{13})|^2, \\ Y(s_{12}, s_{13}) = |\mathcal{M}_{\text{fs}}(s_{12}, s_{13})|^2 - |\bar{\mathcal{M}}_{\text{fs}}(s_{12}, s_{13})|^2, \\ Z(s_{12}, s_{13}) = \text{Im}[\mathcal{M}_{\text{fs}}^*(s_{12}, s_{13}) \bar{\mathcal{M}}_{\text{fs}}(s_{12}, s_{13})]. \quad (6)$$

Here,  $X$ ,  $Y$  and  $Z$  are, respectively, the effective CP-averaged branching ratio, the direct CP asymmetry, and the indirect CP asymmetry. These may be constructed for every point on any Dalitz plot. However, when the final state has a specific flavor, such as in the case of  $B^0 \rightarrow K^+\pi^0\pi^-$ , the quantity  $Z$  has no physical meaning and is therefore left out of our analysis.

In order to obtain the experimental errors on these quantities, we vary the input isobar coefficients over their  $1\sigma$  statistical-uncertainty ranges. We include the correlations between these coefficients when they are given in the papers of Ref. [5].

In addition, note that, since the amplitudes used to construct these observables are fully symmetric under the interchange of the three Mandelstam variables, for any given point on a Dalitz plot there will be five other points where the extracted  $X$ ,  $Y$  and  $Z$  take identical values, and hence do not provide any new information. In order to avoid counting the same information multiple times, we therefore divide each Dalitz plot into six zones by its three axes of symmetry, and use information only from one zone. This is shown in Fig. 1, where we select the dotted zone for our calculations.

The next step is to pick the points on the Dalitz plot where the observables can be evaluated. The idea is to choose the maximum number of points for which the observables evaluated at these points are independent of one another. Ideally, with enough data (and a perfect apparatus), every point in the region of overlap can be treated as an independent source for measuring  $\gamma$ . In practice, however, the maximum number of independent points is limited



**Fig. 1.** Kinematic boundaries and symmetry axes of  $B \rightarrow K\pi\pi$  and  $B \rightarrow KK\bar{K}$  Dalitz plots. The symmetry axes divide each plot into six zones, five of which are marked 2–6. The fifty dots in the region of overlap of the first six zones from all Dalitz plots are used for the  $\gamma$  measurement.

by the number of events observed in the three-body decays. Here we pick a grid with an equal spacing of  $1 \text{ GeV}^2$  between two consecutive points (this spacing is chosen arbitrarily). We find that there are fifty such points. In the experimental data, the process with the smallest statistics is  $B^0 \rightarrow K^0 K^0 \bar{K}^0$ , for which BABAR has reported  $200 \pm 15$  events [5]. Our choice of spacing is consistent with this number of events. We perform a maximum likelihood fit to the observables at these fifty points to obtain  $\gamma$ . Note that this is just an example. Since the final value of  $\gamma$  is essentially the average over all points, its error scales simply as  $1/\sqrt{N}$ . The maximum value that  $N$  can take is limited by the available experimental statistics.

With direct access to the data, a more accurate analysis for determining  $N$  is possible. The data can be separated into bins in each of the two Mandelstam variables; an optimal bin size, not necessarily uniform over the Dalitz plot, can be suitably chosen. Note that it is necessary to choose identical binning for all the processes involved. Observables in each of the  $N$  bins can then be used as an independent source of measuring  $\gamma$ .

Although it is possible in principle to measure both the direct and indirect CP asymmetries in  $B^0 \rightarrow K_S K_S K_S$ , their measurement is currently statistics limited. The experimental Dalitz-plot analysis done by BABAR makes no distinction between the amplitude and its CP conjugate. That is, they take  $A(B^0 \rightarrow K_S K_S K_S) = A(\bar{B}^0 \rightarrow K_S K_S K_S)$ . This has two consequences. First,  $Y$  and  $Z$  vanish for

**Table 1**

Most-likely values of  $\gamma$  (in degrees) extracted from Fig. 2. Results are obtained using the three different fitting methods as explained in the text.

Solution	Fit 1	Fit 2	Fit 3
I	$31^{+2}_{-1}$	$31^{+1}_{-2}$	$32 \pm 2$
II	$77 \pm 2$	$78 \pm 2$	$77 \pm 2$
III	$261^{+2}_{-3}$	$259^{+3}_{-2}$	$259^{+2}_{-3}$
IV	$314 \pm 2$	$315 \pm 2$	$315 \pm 2$

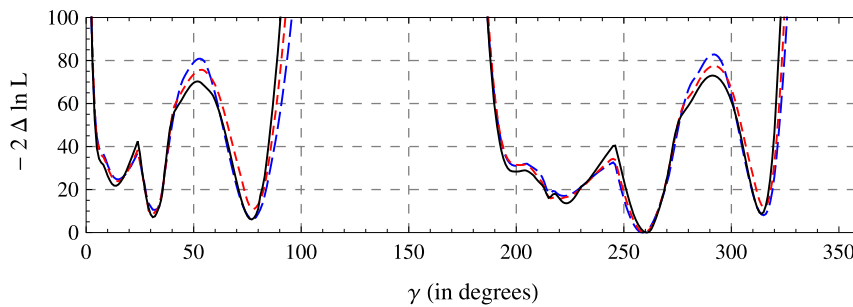
every point of the Dalitz plot. Second, this requires that  $\tilde{P}'_{uc}$  be set to zero in Eq. (4). The removal of an equal number of unknown parameters (amplitude and phase of  $\tilde{P}'_{uc}$ ) and observables does not affect the viability of the method described above.

With the observables in hand, we now perform a maximum likelihood analysis for extracting  $\gamma$ . For each of the fifty points in region 1 of Fig. 1, we construct the  $-2\Delta \ln L(\gamma)$  function, where  $L$  represents the likelihood, which we then minimize over all the hadronic parameters for that point. Since we have assumed the observables of Eq. (6) to be uncorrelated, the fifty points are independent, so that the sum of log-likelihood functions over all points gives us a joint likelihood distribution. The local minima of this function are then identified as the most-likely central values of  $\gamma$ . The values of  $\gamma$  for which there is a unit shift along the vertical axis of the  $-2\Delta \ln L(\gamma)$  vs  $\gamma$  plot represent the  $1\sigma$  range corresponding to each central value.

We perform the likelihood maximization in three different ways and plot our results in Fig. 2. We first consider the scenario in which flavor SU(3) is a good symmetry. That is, we fix  $|\alpha_{SU(3)}| = 1$ ; our analysis involves only the four  $B^0$  decay channels. The most-likely values of  $\gamma$  obtained in this way are listed under Fit 1 in Table 1.

Second, we allow for SU(3) breaking and treat it as follows. We compare the Dalitz plots for the two processes  $B^+ \rightarrow K^+ \pi^+ \pi^-$  and  $B^0 \rightarrow K^+ K^0 K^-$  point by point. Theoretically, the amplitudes for these processes differ only by the parameter  $\alpha_{SU(3)}$ . The ratio of  $X$ 's constructed from the two Dalitz plots then gives us  $|\alpha_{SU(3)}|^2$ . (Note that a similar ratio constructed from the  $Y$ 's has an enormous error due to the smallness of  $Y$ . We are therefore unable to extract any interesting physical information from such a ratio.) Averaged over the fifty points we find  $|\alpha_{SU(3)}| = 0.97 \pm 0.05$ . This shows that, on average, SU(3) breaking is small. We use  $|\alpha_{SU(3)}|$  found in this way to correct the observables from the  $B \rightarrow KK\bar{K}$  Dalitz plots and use the corrected numbers in a new maximum-likelihood analysis for finding  $\gamma$ . We present the results under Fit 2 in Table 1.

In the third maximum-likelihood analysis, we consider observables from all five Dalitz plots but now include  $|\alpha_{SU(3)}|$  as an additional unknown hadronic parameter. The results from this method are listed under Fit 3 in Table 1.



**Fig. 2.** Results of maximum-likelihood fits. The solid (black) curve represents the fit assuming flavor-SU(3) symmetry. The short dashes (red) represent the fit where flavor-SU(3) breaking is fixed by a point-by-point comparison of Dalitz plots for  $B^+ \rightarrow K^+ \pi^+ \pi^-$  and  $B^0 \rightarrow K^+ K^0 K^-$ . The long dashes (blue) represent the fit with inputs from five Dalitz plots and an extra hadronic fit parameter  $|\alpha_{SU(3)}|$ . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The maximum-likelihood analysis indicates that, in each of the three methods described above, the data favor four distinct discretely-ambiguous values of  $\gamma$ . (Due to the fact that the fit involves nonlinear equations, it is not surprising to find multiple solutions for  $\gamma$ .) In Table 1 we present the most-likely values of  $\gamma$  extracted using these three methods. It is evident from the results that the inclusion of an SU(3)-breaking parameter  $|\alpha_{\text{SU}(3)}|$  shifts the preferred values of  $\gamma$  by only a tiny amount. This indicates that the leading-order effects of flavor-SU(3) breaking are well under control in three-body  $B$  decays. While one cannot completely remove this source of theoretical error from our analysis, the uncertainties are rather small.

Even though there are four preferred values of  $\gamma$ , in all cases the error is small, 2–3°. Although this may seem surprising at first sight, it really is not when one remembers that there are, in fact, fifty independent measurements of  $\gamma$ . Roughly speaking, if each measurement has an error of  $\pm 20^\circ$  [6], which is somewhat larger than other methods, then when we take a naive average, we divide the error by  $\sqrt{50}$ , giving a final error of  $\sim 3^\circ$ . And, as noted above, if the number of independent points in the Dalitz plot is not fifty, but twenty (for example), the error will be increased by about  $\sqrt{50}/\sqrt{20}$ .

Because the observables at different points are all computed using the same isobar coefficients, there is a certain level of correlation, reducing the degree to which these points are independent. That is, the effective value of  $N$  is decreased, leading to an increased error on  $\gamma$ . We refer to this as the “correlation error.” A precise estimate of the correlation error requires detailed information about the statistical and systematic errors on the isobar parameters, as well as the correlations between them. Even if such information were completely available, a full analysis would involve a multi-parameter fit requiring computational power that is well beyond the scope of our present analysis.

There are other sources of error that have not been included in our (simple) analysis. First, and most importantly, all errors considered to this point have been entirely statistical – the systematic error has not been included. The reason is that only statistical errors were given for the isobar coefficients in the BABAR papers of Ref. [5]. Second, we have only taken leading-order flavor-SU(3) breaking into account. Higher-order flavor-SU(3) breaking may arise due to the nonzero mass difference between pions and kaons, and between intermediate resonances. This said, the error due to leading-order SU(3) breaking is evidently small. It is unlikely that the error due to higher-order SU(3) breaking is larger.

To summarize: we have demonstrated that it is possible to cleanly extract  $\gamma$  from  $B \rightarrow K\pi\pi$  and  $B \rightarrow KK\bar{K}$  decays, and we find four most-likely values. Three of these –  $32^\circ$ ,  $259^\circ$ , and  $315^\circ$

– are in disagreement with the SM (is this a “ $K\pi\pi$ – $KK\bar{K}$  puzzle”?). However, one solution –  $77^\circ$  – is consistent with the SM. In all cases, although we find a small error, we have made a number of assumptions about the data in performing the analysis, and several sources of error have been ignored. The full error on  $\gamma$  must be determined in order to judge the efficacy of this method. This can only be done with direct access to the data, and hopefully this procedure for extracting weak-phase information from three-body  $B$  decays will be incorporated into the programs of future experiments (e.g., a super  $B$  factory, perhaps LHCb).

## Acknowledgements

A special thank you goes to E. Ben-Haim for his important input to this project. We also thank J. Charles, M. Gronau, N. Rey-Le Lorier, J. Rosner and J. Smith for helpful communications. BB would like to thank G. Bell and WG IV of CKM 2012. This work was facilitated in part by the workshop *New Physics from Heavy Quarks in Hadron Colliders*, which was sponsored by the University of Washington and supported by the DOE under contract DE-FG02-96ER40956. This work was financially supported by NSERC of Canada (BB, DL) and by FQRNT du Québec (MI).

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